TDP (Honours) 5th Semester Exam., 2022

PHYSICS

(Honours)

SECOND PAPER : DSE – II

Full Marks : 80 *Time* : 3 Hours

Answer from ***both*** *the Sections as directed.*

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

SECTION—A

1. Answer *any* **eight** of the following questions : 2×8=16

(a) Given, $A = \begin{bmatrix} 1 & -1 & -1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

Find AB.

(b) What do you mean by vector space?

(c) Define singular and non-singular matrix.

(d) State and explain quotient law.

(e) The set of vectos is given as $\{(a, b)(\mathbb{R}^2 : b = 4a + 1\}$. Identify if the given set is a vector space or not.

(f) Is it possible to diagonalyze the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$? Give the reason.

(g) Is the inverse element of an element in a group is unique? Justify logically.

(h) A subset S of G is a subgroup of a group $(G, *)$; then show that for any pair of elements $a, b \in S$, $a * b^{-1} \in S$

(i) Two dice are tossed. Find the probability so that the total score is a prime number.

(j) What is the utility of different statistical distribution?

SECTION—B

There are *four* questions from Question No. 2 to Question No. 5. Answer either *(a)* **or** *(b)* from each question given below : 16×4=64

2. (a) (i) Show that eigenvectors of a Hermitian matrix corresponding to two distinct eigenvalues are orthogonal.

(ii) Solve the given set of equation

$$x + z = 4$$
$$2x + y + z = 7$$
$$x + y + 27 = 9$$

(iii) Find the eigenvalue and eigenvector of the given matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{bmatrix}$$

(iv) Determine if the given matrix is singular or not. If it is non-singular, find the inverse :

$$\begin{bmatrix} 6 & 0 \\ -3 & 2 \end{bmatrix}$$

4+4+4+4=16

(OR)

(b) (i) Define a singular matrix. For what value of x, the matrix A is singular

where $A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$

What is the physical significance of singular matrix?

(ii) Verify Cayley-Hamilton theorem for the matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and find B^{-1}.

(iii) What does it physically mean that the two matrices are orthogonal?

If λ is an eigenvlaue of an orthogonal matrix A, then show that $1/\lambda$ is also eigenvalue of the same matrix A.

(iv) Show that the transpose of the product of two matrices is the product of their transposes in reverse order.

(1+3+1)+4+(1+3)+3=16

3. (a) (i) What do you mean by abstruct binary operation?

(ii) Let $V = \mathbb{R}^+$ be a set of all positive reals. The operation addition in the set is defined as $\vec{x} + \vec{y} = \vec{x}.\vec{y}$ where $x, y \in \mathbb{R}^+$. The scalar multiplication by a scalar K is defined as $K\vec{x} = x^K \quad x \in \mathbb{R}$.
Show that V is a vector space.

(iii) If V is a vector space and u_1 and u_2 are two subspaces. Find the common point in the intersection of two subspaces. Identify if the vectors V_1, V_2 and V_3 are linealy dependent or not where $V_1 = (1, -2, 3)$, $V_2 = (5, 6, -1)$ and $V_3 = (3, 2, 1)$.

(iv) Show that the set $B = \{u_1, u_2\}$ is a basis vector for $\mathbb{R}^2$ where $u_1 = (3, 4)$ and $u_2 = (1, -3)$. 2+4+2+4+4=16

(OR)

(b) (i) Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one.

(ii) Given $A(r, s, t)B^{st} = C^r$,

In the above relation C^r is a tensor B^{st} is also an arbitrary tensor. Then prove that $A(r, s, t)$ is also a tensor and can be represented as A^r_{st}.

(iii) Let $\vec{V} = \mathbb{R}^3$ and $B = \left\{\vec{i}, \vec{j}, \vec{k}\right\}$ where $\vec{i} = (2, 0, 0)$, $\vec{j} = (0, 2, 0)$, and $\vec{k} = (0, 0, 2)$. Identify if B is a linearly independent set in $\mathbb{R}^3$ or not. Can B be considered as basis for $\mathbb{R}^3$?

(iv) Prove that the contraction of tensor A_q^p is a scalar or invariant.

(v) Prove that $\varepsilon_{ilm} \cdot \varepsilon_{jlm} = 2\delta_{ij}$.

3+3+(3+1)+3+3=16

4. (a) (i) What is subgroup? What are the two trivial subgroups of a group?

(ii) Define homomorphism and isomorphism between two algebraic systems. Given an example of homomorsphism.

(iii) Show that in a group $(G, *)$ the equations $a * x = b$ and $y * a = b$ have unique solutions for the unknown x and y, given by $x = a^{-1} * b$, $y = b * a^{-1}$, when $a, b \in G$.

(iv) Let Z be the group of integers with binary operation $*$ defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of group $\{z, *\}$. 2+2+3+3+3+3=16

(OR)

(b) (i) Obtain the direct products of all the irreducible representation of the group C_{3u} and reduce them into direct sum of the irreducible representation.

[Continued]

12+8+16+4 = 40

(ii) Prove that the elements common to two intersecting subgroups G_1 and G_2 from a subgroup of group G.

(iii) Prove that the element of the two different classes are all distinct. A group comprises entire class.

(iv) Write the group multiplication table of any group and explain their importance of it. 4+4+3+2+3=16

5. (a) (i) Show that the probability of the two event and their combined probability are connected by the following relation :

$$P(A+B) = P(A) + P(B) - P(AB)$$

(ii) Show that if two events are independent, the probability of both occurring is equal to the product of the probability of each event occurring independently.

(iii) Given, $P(A) = 1/2$ and $P(A \cup B) = 3/5$. Find the value of P(B) if the event A and B are mutually exclusive and independent. What do you mean by mutually independent and exclusive?

(iv) If A and A' are two complimentary events, then show that the probability of the event is equal to the difference of one and the other event.

4+4+(4+1)+3=16

[Turn Over]

(b) (i) What do you mean by binomial law of probability?

(ii) X follows a binomial distribution with parameters $n = 8$ $P = 1/2$. Then find

$P(|x-4|) \leq Z$.

(iii) What do you mean by repeated trial? Calculate the probability that an event happens at least m times in n trials.

(iv) Let A and B be independent event with $P(A) = \cdot 3$ and $P(B) = \cdot 4$. Find

$P(A \cap B)$ and $P(A \cup B)$. 4+4+4+4=16